EXERCISE SHEET 4: KINEMATIC MODEL OF SELF-REPLICATION AND CELLULAR AUTOMATA

Exercise 1 (Elementary logical elements in von Neumann's kinematic model)**.** *Let* $\wedge, \vee, \neg$ *denote the logical operations* AND, OR, NOT *respectively. We define the binary logical* NAND *operation as* $\mathrm{NAND}(x, y) = \neg(x \wedge y)$*. Show that you can express all of the operations below just in terms of the* NAND *operation.*

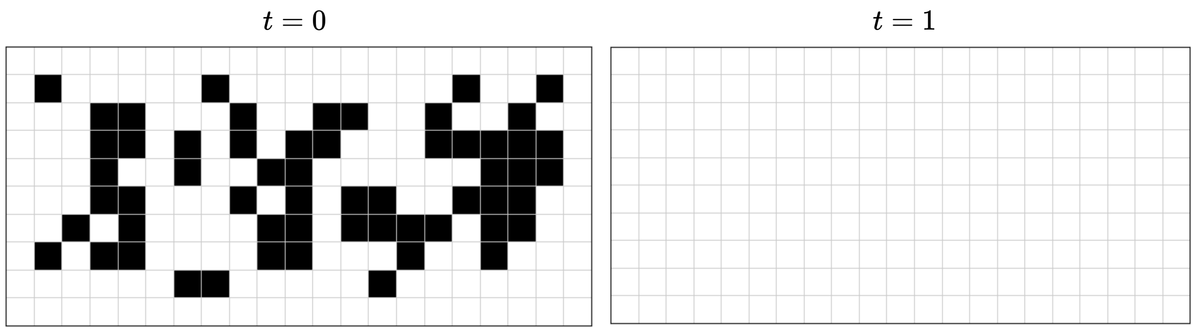
$$\neg x \qquad x \wedge y \qquad x \vee y$$

Therefore, we can reduce the number of elementary components in von Neumann's kinematic model from eight to four. Why do you think von Neumann nevertheless chose to include the other logical gates?

Exercise 2 (Simulating Game of Life)**.** *Game of Life is a two-dimensional cellular automaton with Moore neighbourhood and states* $S = \{0, 1\}$*. Typically, we interpret a cell in state 0 as "dead" and a cell in state 1 as "alive". The local update rule is defined as follows:*

- *If the central cell is alive, and has exactly two or three alive neighbours, it stays alive.*
- *If the central cell is dead, and has exactly three alive neighbours, it becomes alive.*
- *Otherwise, the cell will always becomes (or stays) dead.*

Compute one iteration on Game of Life on the following configuration.



Exercise 3 (CA Iterations)**.** *Let* $\mathcal{A} = (S^{\mathbb{Z}}, F)$ *be a 1D CA with local rule* f *and neighbourhood* $N = (-2, -1, 0, 1, 2)$*, and let* $k \in \mathbb{N}$*. Show that the dynamical system* $\mathcal{A}^k = (S^{\mathbb{Z}}, F^k)$ *(which iterates* F k*-times) is also a 1D CA. What is its neighbourhood?*

Exercise 4 (Quiescent state)**.** *Let* $\mathcal{A} = (S^{\mathbb{Z}}, F)$ *be a 1D CA with local rule* f*. We say that a state* $q \in S$ *is quiescent, if* $f(q, q, \ldots, q) = q$*. Show that there exists a* $k \in \mathbb{N}$ *such that* $\mathcal{A}^k$ *always has at least one quiescent state.*

Exercise 5 (Von Neumann CA Logic Gates)**.** *Using the ordinary transmission signals and confluent states, construct a pattern in the von Neumann CA which implements the following Boolean functions?*

(a) $x \vee y$ *(b)* $(x \vee y) \wedge (x \vee z)$

Exercise 6 (Von Neumann CA Clocks)**.** *A clock with period* $t \in \mathbb{N}$ *in the von Neumann CA is a pattern with a single outgoing channel of OTs, which produces an active pulse every* t *time-steps. Can you design such a pattern using the ordinary transmission signals and confluent states?*

Exercise 7 (General Pulser)**.** *Given the example seen in the lecture, describe the construction of* $\mathrm{P}(c)$ *for a general sequence* $\{0,1\}^+$.

Exercise 8 (Decoder Output)**.** *What is the output of* $\mathrm{D}(1010)$ *for the input sequence* 1110*?*

Exercise 9 (General Decoder)**.** *Using the example seen in the lecture, describe a construction of* $\mathrm{D}(c)$ *for a general sequence* $c \in \{0,1\}^+$ *containing at least one* 1*.*

Exercise 10 (General Recognizer)**.** *Using the example seen in the lecture, describe a construction of* $\mathrm{R}(c)$ *for a general sequence* $c \in \{0,1\}^+$ *containing at least one* 1*.*