

EXERCISE SHEET 5: VON NEUMANN'S CA SELF-REPLICATOR

Exercise 1 (Decoder Output). *What is the output of $D(1010)$ for the input sequence 1110?*

Exercise 2 (General Decoder). *Using the example seen in the lecture, describe a construction of $D(c)$ for a general sequence $c \in \{0, 1\}^+$ starting with a 1.*

Exercise 3 (General Recognizer). *Using the example seen in the lecture, describe a construction of $R(c)$ for a general sequence $c \in \{0, 1\}^+$ containing at least one 1.*

Exercise 4 (Construction Arm Orders I). *Work out a sequence of higher-level steps, similar to those you have seen in the lecture, which results in the construction arm retracting one step to the left.*

Exercise 5 (Construction Arm Orders II). *Define a sequence of pulses that should be sent via the O and S channels, so that the construction arm creates an unexcited confluent state and then retracts one step to the left.*

Exercise 6 (Destroying a Tape Symbol). *We first send instructions through the bottom construction arm that change the rightmost return channel state \leftarrow into \uparrow , resulting in the configuration shown in Figure 1(b). Next, we send the sequence 1100001 through the S_B channel at time t and through the O_T channel at time $t + 1$ (so that both signals reach the tape symbol simultaneously). Verify that the resulting configuration is always the one shown in Figure 1(c).*

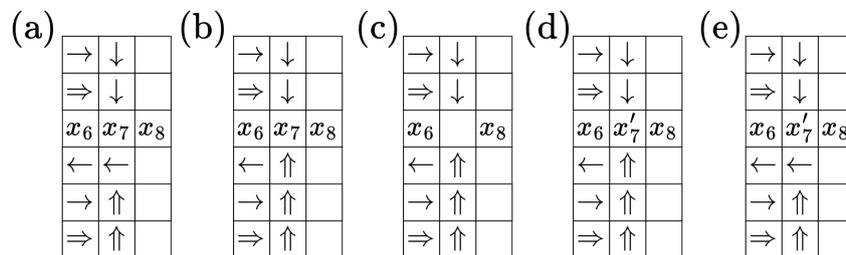


Figure 1: The process of writing a new tape symbol x'_7 .

Exercise 7 (Coarse-graining the self-replicator). *The resulting von Neumann self-replicator is estimated to span between 50,000 and 200,000 cells. Can you think of a clever way to define a different CA (possibly with many more states) that also contains a universal self-replicator based on the exact same design by von Neumann but spans roughly just 1/4 (1/8, 1/16, ...) of cells?*

Exercise 8. *Let $\mathcal{T} = (Q, \Sigma, \delta)$ be a Turing machine. Define a one-dimensional cellular automaton which realizes the computational process defined by \mathcal{T} .*

Hint: For instance, consider the CA states to be of the form $\Sigma' \times (Q \cup \{\emptyset\})$; thus, each cell carries the information about a tape symbol, and whether the head is currently reading this symbol, and what state is it in.